SAMPLE QUESTIONS

The following questions are similar to those in the test. They illustrate the range of the actual test in terms of the subject-matter areas tested and the difficulty of the questions posed. An answer key appears after the sample questions.

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case select the one that is the best of the choices offered.

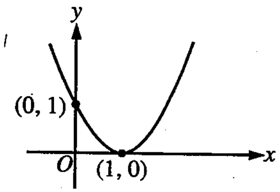

Figure 1

1. Which of the following could be an equation of the curve shown in Figure 1 ?

   (A) $y = x^2 + 1$ (B) $y = (x + 1)^2$ (C) $y = |x - 1|$
   (D) $y = (x - 1)^2$ (E) $y = |x| + x + 1$

2. What is the least upper bound of the set of all numbers A such that a polygon with area A can be inscribed in a semicircular region of radius 1 ?

   (A) $\frac{4}{5}$ (B) $\frac{2}{\sqrt{5}}$ (C) 1 (D) $\frac{\pi}{2}$ (E) 2

3. A certain procedure requires 2^{60} calculations. A programmer decides that, to be useful on his computer, the procedure must be modified so as to require only 2^{20} calculations. If the procedure is modified each day in such a manner that the number of calculations required is exactly one-half that required by the procedure on the previous day, what is the total number of days that will be required to obtain a modification using only 2^{20} calculations?

   (A) 1 (B) 2 (C) 3 (D) 30 (E) 40

4. The solution set for the equation $\begin{vmatrix} 2 & 3 & x \\ 2 & 1 & x^2 \\ 6 & 7 & 3 \end{vmatrix} = 0$ is

   (A) the empty set (B) $\{0\}$ (C) $\{1\}$ (D) $\{1, -3\}$ (E) $\{\sqrt{3}, -\sqrt{3}\}$

5. If $f(x) = x|x|$ for all real numbers x, then $f'(x)$ is a real number for

   (A) no real number x
   (B) $x = 0$ only
   (C) $x > 0$ only
   (D) $x \neq 0$ only
   (E) all real numbers x

6. If f is a function having derivatives of all orders and if $f'(x) = (f(x))^2$, then the nth derivative of f at x is given by

   (A) $n(f(x))^n$ (B) $n!(f(x))^{n+1}$ (C) $(n + 1)!(f(x))^{n+1}$
   (D) $(n + 1)(f(x))^n$ (E) $n(f(x))^{2n}$

7. Let p and q be constants. If $f(x) = p \sin x + qx \cos x + x^2$ for all real numbers x and if $f(2) = 3$, then $f(-2)$ is

   (A) -3 (B) -1 (C) 1 (D) 5
   (E) not uniquely determined by the information given

8. Let f be a continuous function such that for all $c > 0$,

   (i) $f(c) > 0$ and
   (ii) the region bounded by the line $x = c$, by the coordinate axes, and by the curve with equation $y = f(x)$ has area ce^c.

   For all $x > 0$, $f(x) =$

   (A) e^x (B) xe^x (C) $xe^x - e^x$ (D) $xe^x + e^x$ (E) $x^2e^x - xe^x$

9. If the binary operation $*$ is defined for all integers by $a * b = a + b - ab$, then the properties of $*$ include which of the following?

I. $*$ is commutative.
II. $*$ is associative.
III. There exists some integer that is an identity for $*$.

(A) I only (B) III only (C) I and II only
(D) I and III only (E) I, II, and III

10. Which of the following is an equation of the curve through the origin that intersects at right angles all the curves satisfying the differential equation $\frac{dy}{dx} = x + 1$?

(A) $e^{-y} = x + 1$ (B) $e^{y} + x - 1 = 0$ (C) $e^{y} = x + 1$
(D) $2y = x^2 + 2x$ (E) $y(x^2 + 2x) = -2$

11. If x and y are integers such that $x \geq 3$ and $x - y \geq 9$, then the minimum possible value of $3x + 7y$ is

(A) -33 (B) 0 (C) 9 (D) 27 (E) nonexistent

12. The interval of convergence of the series

$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^3 2^{n+2}}$ is indicated by

(A) $-1 < x < 1$ (B) $-1 \leq x \leq 1$ (C) $-1 < x < 3$
(D) $-1 \leq x \leq 3$ (E) $0 \leq x \leq 2$

13. Let f be a continuous function of x (f not identically zero) and let s and t be nonzero numbers. If $I = t\int_0^{\frac{s}{t}} f(s + tx)\,dx$, then the value of I

(A) varies with x
(B) depends on the ratio $\frac{s}{t}$
(C) depends on s, but is independent of t and x
(D) depends on t, but is independent of s and x
(E) is constant for all s, t, and x

14. Let x and y be integers such that $9x + 5y$ is divisible by 11. For which of the following values of k must $10x + ky$ be divisible by 11 ?

(A) 0 (B) 1 (C) 3 (D) 7 (E) 8

15. If T is the linear transformation mapping the vectors (1,0,0), (0,1,0), and (0,0,1) to the vectors (2,2,1), (1,1,2), and (0,0,1), respectively, which of the following is the image of the vector (2,3,4) under T ?

(A) (7,7,12) (B) (7,7,11) (C) (7,7,8) (D) (6,6,11) (E) (3,3,4)

16. If f is infinitely differentiable everywhere, then

$$\lim_{k\to 0}\left[\lim_{h\to 0} \frac{f(p+k+h) - f(p+k) - f(p+h) + f(p)}{hk}\right] =$$

(A) $f'(p)$ (B) $f''(p)$ (C) $(f'(p))^2$ (D) $f'(f'(p))$ (E) $f'(h)f'(k)$

17. Alternately a fair coin is tossed and a fair die is thrown, beginning with the coin. What is the probability that the coin will register a "head" before the die registers a "5" or a "6" ?

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{3}{5}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

18. If $f(x) = x^{\left(\frac{1}{x-1}\right)}$ for all positive $x \neq 1$ and if f is continuous at 1, then $f(1)$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e
(E) none of the above

19. Suppose that H is a nonempty subset of a multiplicative group G and that H is closed under the group operation. Which of the following conditions is NOT sufficient to ensure that H is a subgroup of G ?

(A) The identity element of G is in H.
(B) Every element of H has a unique inverse in H.
(C) If $x, y \in H$, then $xy^{-1} \in H$.
(D) If $x, y \in H$, then $x^{-1}y^{-1} \in H$.
(E) If $y \notin H$, then $y^{-1} \notin H$.

20. Let A and B be topological spaces, let f be a mapping from A to B, and let f^{-1} be the inverse of f. Under which of the following conditions must f be continuous?

(A) The image under f of any open set in A is an open set in B.
(B) The image under f of any closed set in A is a closed set in B.
(C) The image under f of any bounded set in A is a bounded set in B.
(D) The image under f^{-1} of any open set in B is an open set in A.
(E) The image under f^{-1} of any discrete set in B is a discrete set in A.

21. In the process of constructing a highway across a certain region in which there are many hills and valleys, the engineer will be certain that

{ there is some level in between the elevations of the highest hill and the lowest valley at which the surface of the highway can be laid using the tops of the hills as fill material for the valleys and such that no additional fill dirt need be brought in from another region and none will be left to be hauled away. }

To build a mathematical model of this situation, let S be a long, narrow rectangular region (the roadbed) bounded by the lines $x = a$, $x = b$, $y = c$, and $y = d$; let f be a continuous function on S with M and m being, respectively, the maximum and minimum values of f on S. If the graph of f is identified with the surface of the land, then, of the following, which best corresponds to the assertion set off in braces above?

(A) There exists a point p in S such that $m \le f(p) \le M$.

(B) There exists a value q of f such that $M - m = q$.

(C) There exists a point p in S such that $\iint_S f = f(p) \cdot$ (area of S).

(D) $\int_a^b \left(\int_c^d f(x, y)dy \right) dx = \left.\frac{\partial f}{\partial x}\right|_{(a, b)} + \left.\frac{\partial f}{\partial y}\right|_{(c, d)}$

(E) There exists a value q of f such that $\frac{q}{(\text{area of } S)} = \left.\frac{\partial f}{\partial x}\right|_{(a, b)} + \left.\frac{\partial f}{\partial y}\right|_{(c, d)}$.

22. Consider the following algorithm.

Step 1. Set $h = k = 2$
Step 2. Set $s_1 = 2$
Step 3. If $k > 100$ then stop
Step 4. Increase h by 1
Step 5. If h is NOT divisible by any of $s_1^2, \ldots, s_{k-1}^2$ then set $s_k = h$ and increase k by 1
Step 6. Go to step 3

The sequence $\{s_k\}$ produced by this algorithm is the sequence of

(A) prime numbers less than 100
(B) the first 100 prime numbers
(C) the integers n such that $1 < n < 100$ and no square of an integer except 1 divides n
(D) the first 100 integers $n > 1$ such that no square of an integer except 1 divides n
(E) the integers n such that $1 < n < 100$ and n is not the square of an integer

23. Each of two sets of data, D_1 and D_2, is divided into categories, C_1 and C_2, and the following table is devised to record the number of data in each category.

	C_1	C_2
D_1		
D_2		

Furthermore, there are two systems of weights, W_1 and W_2, each of which assigns a score for each datum in each category, and the following table is devised to show the score assigned to each category under each system of weights.

	W_1	W_2
C_1		
C_2		

Let x_{ij} be the number of elements of D_i that are put into category C_j. Let y_{ij} be the weight assigned to C_i under the system W_j. If $X = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix}$ and $Y = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}$, which of the following would be the matrix of entries showing the total score of set D_i under W_j ?

(A) $X + Y$ (B) XY (C) YX (D) XYX^{-1} (E) YXY^{-1}

24. Of the following equations, which has the greatest number of roots between 100 and 1,000 ?

(A) $\sin x = 0$ (B) $\sin (x^2) = 0$ (C) $\sin \sqrt{|x|} = 0$
(D) $\sin (x^3) = 0$ (E) $\sin \sqrt[3]{x} = 0$

25. If $f(x) = \begin{cases} \dfrac{x}{|x|} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$, then $\int_{-1}^{3} f(x)\,dx =$

(A) $-e$ (B) $-\log 2$ (C) 1 (D) 2 (E) e

26. Two planes P and Q intersect at an angle of 30°. Two distinct rays, one in P and one in Q, form an angle of $x°$ with its vertex in the intersection of P and Q. What is the least possible value of x ?

(A) 30 (B) 36 (C) 45 (D) 90 (E) There is no least possible value of x.

27. If f is a continuous real-valued function with domain a closed interval $[a, b]$ such that $f'(x) = 0$ for one and only one number x between a and b, then f

(A) might not have a maximum on $[a, b]$
(B) cannot have an even number of extrema on $[a, b]$
(C) cannot have a maximum at one endpoint and a minimum at the other
(D) might be monotonically increasing
(E) might be unbounded

28. Which of the following is equal to the product $P^{-1}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}P$ for some invertible 2×2 matrix P ?

(A) $\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$ (B) $\begin{pmatrix} 2 & 2 \\ 3 & 2 \end{pmatrix}$ (C) $\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$ (D) $\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$ (E) $\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

29. Which of the following correctly describe the graph G in the xy-plane of $y = \frac{1}{x}\sin\frac{1}{x}$?

I. G does not intersect any line $y = c$, where $c < 0$.
II. G contains infinitely many points $(x, 0)$ with $0 < x < 1$.
III. G contains no points $(x, 0)$ with $1 < x$.

(A) I only (B) II only (C) III only (D) I and III (E) II and III

30. The order of the element $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 5 & 1 & 3 & 2 \end{pmatrix}$ of the symmetric group S_5 is

(A) 2 (B) 3 (C) 6 (D) 8 (E) 12

31. If f is a continuous, decreasing function defined on the positive reals and its graph is concave upward, then the graph of the inverse function f^{-1} is

(A) decreasing and concave upward
(B) decreasing and concave downward
(C) increasing and concave upward
(D) increasing and concave downward
(E) not necessarily any of the above

32. How many different solutions, modulo 24, does the congruence $18x \equiv 12 \pmod{24}$ have?

(A) None (B) One (C) Three (D) Six (E) Twelve

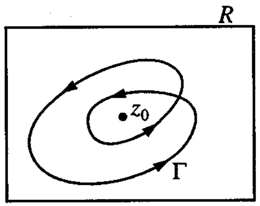

Figure 2

33. Let f be analytic in a region R with the single exception of point z_0. If the residue of f at z_0 is 1, what is the value of $\int_\Gamma f(z)dz$, where Γ is the path shown in Figure 2 ?

(A) 0 (B) 1 (C) 2 (D) $2\pi i$ (E) $4\pi i$

34. Let f be a function with domain $[-1, 1]$ such that the coordinates of each point (x, y) of its graph satisfy $x^2 + y^2 = 1$. The total number of points at which f is necessarily continuous is

(A) zero (B) one (C) two (D) four (E) infinite

35. If $A = \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, which of the following matrices is zero?

(A) $A^2 - A - 5I$ (B) $A^2 + A - 5I$ (C) $A^2 + A - I$
(D) $A^2 - 4I$ (E) $A^2 - 3A + 5I$

Answer Key

1. D	8. D	15. A	22. D	29. E
2. D	9. E	16. B	23. B	30. C
3. E	10. A	17. E	24. D	31. A
4. D	11. E	18. D	25. D	32. D
5. E	12. D	19. A	26. E	33. E
6. B	13. C	20. D	27. D	34. C
7. D	14. E	21. C	28. D	35. A

NOTES